P1 Chapter 6 :: Circles

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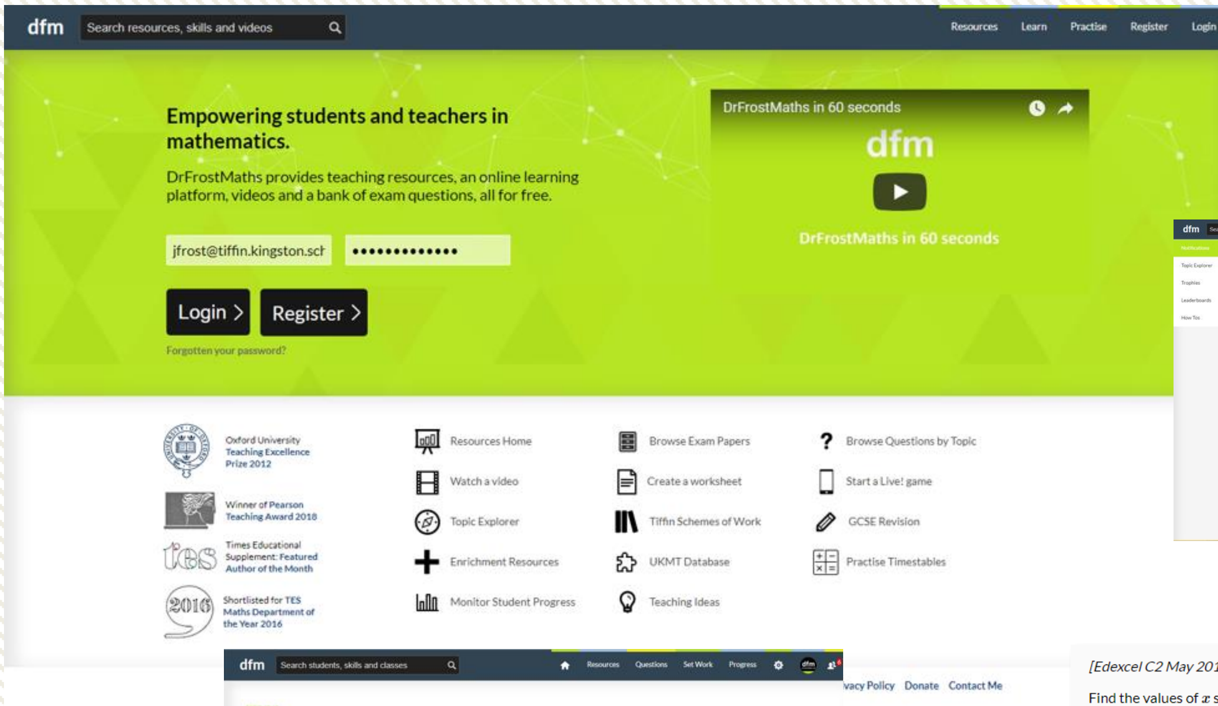

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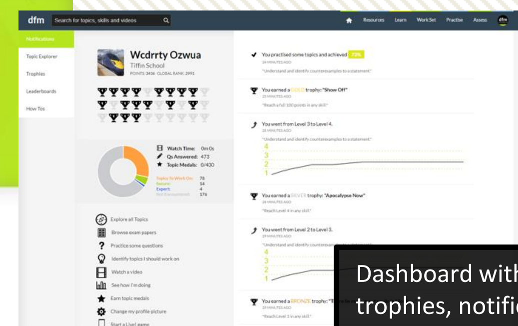

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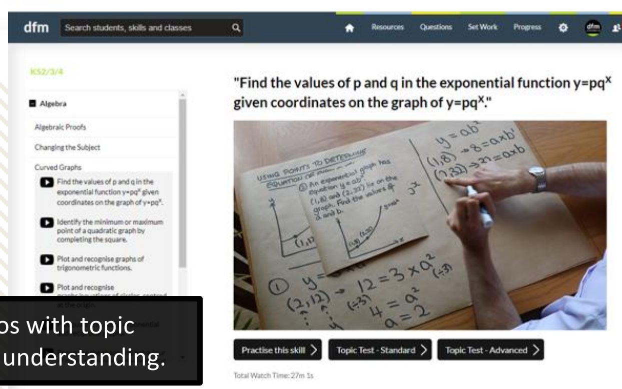
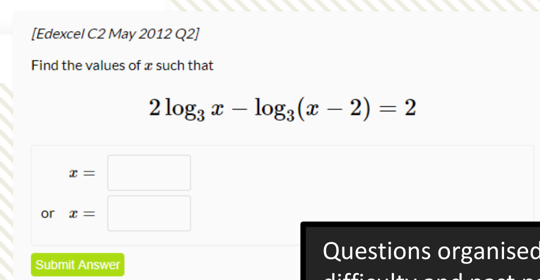


Teaching videos with topic tests to check understanding.



Questions organised by topic, difficulty and past paper.

Chapter Overview

From this chapter onwards, the majority of the theory you will learn is new since GCSE.

1:: Equation of a circle

The diameter of a circle is AB where A and B have the coordinates $(2,5)$ and $(8,13)$. Determine the equation of the circle.

2:: Intersections of lines + circles

Show that the line $y = x - 7$ does not meet the circle $(x+2)^2 + y^2 = 33$

NEW! since GCSE
You should already know the equation $x^2 + y^2 = r^2$ for a circle centred at the origin, but not a circle centred at a specified point.

3:: Chords, tangents and perpendicular bisectors.

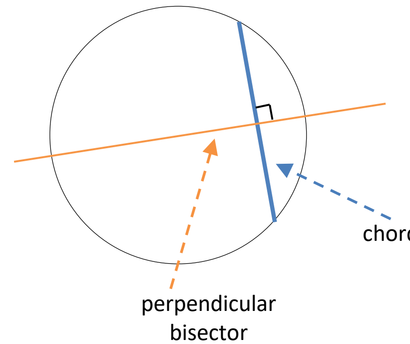
A circle C has the equation $(x-5)^2 + (y+3)^2 = 10$. Find the equations of the two possible tangents whose gradient is -3.

4:: Circumscribing Triangles

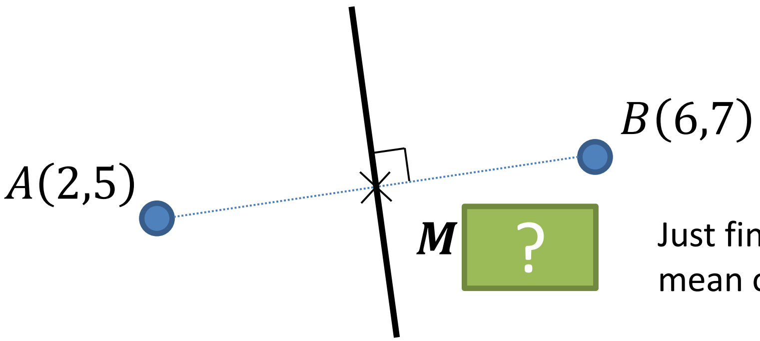
Find the equation of the circle that passes through the points $A(-8,1), B(4,5), C(-4,9)$.

Midpoints and Perpendicular Bisectors

Later in the chapter you will need to find the perpendicular bisector of a chord of a circle.



What two properties does a perpendicular bisector of two points A and B have?



Just find the mean of the x values and the mean of the y values.

1. It passes through the midpoint of AB.

2. It is perpendicular to AB.

Equation?

?

Test Your Understanding

Find the perpendicular bisector of the line AB where A and B have the coordinates:

a) $A(4,7), B(10,17)$

b) $A(x_1, y_1), B(x_2, y_2)$

a

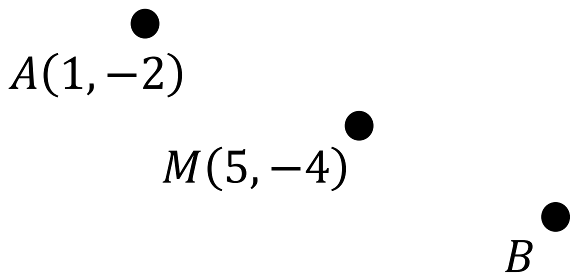
?

b

?

Fro Note: Do <u>not</u> try to memorise this!

A line segment AB is the diameter of a circle with centre $(5, -4)$. If A has coordinates $(1, -2)$, what are the coordinates of B?

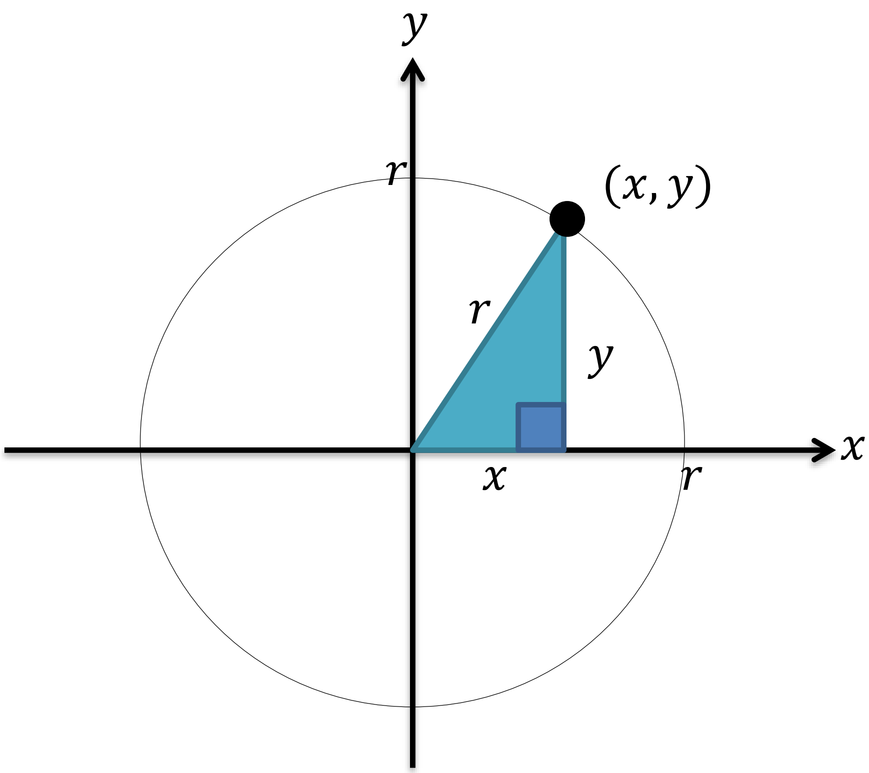


?

Exercise 6A/6B

Pearson Pure Mathematics Year 1/AS
Page 115, 116-117

Equation of a circle

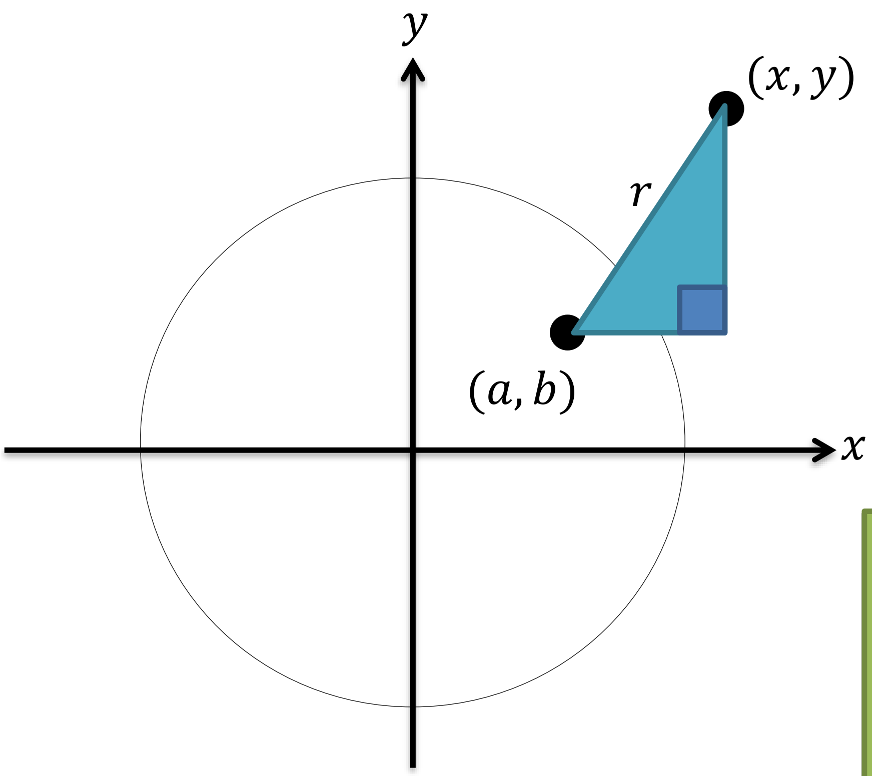


Recall that a line can be a set of points (x, y) that satisfy some equation. Suppose we have a point (x, y) on a circle centred at the origin, with radius r. What equation must (x, y) satisfy?

(Hint: draw a right-angled triangle inside your circle, with one vertex at the origin and another at the circumference)

?

Equation of a circle



Now suppose we shift the circle so it's now centred at (a, b). What's the equation now?

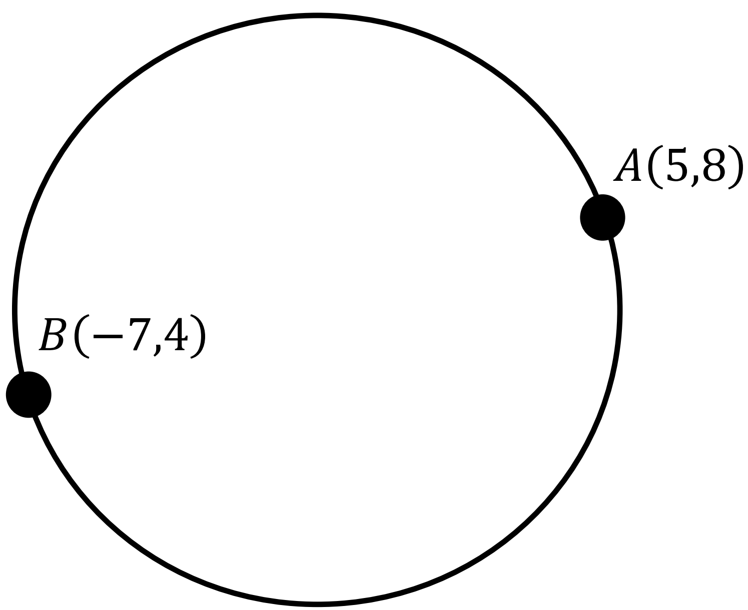
(Hint: What would the sides of this right-angled triangle be now?)

?

Quickfire Questions

Centre	Radius	Equation
$(0,0)$	5	?
$(1,2)$	6	?
?	?	$(x+3)^2+(y-5)^2=1$
?	?	$(x+5)^2+(y-2)^2=49$
?	?	$(x+6)^2+y^2=16$
?	?	$(x-1)^2+(y+1)^2=3$
?	?	$(x+2)^2+(y-3)^2=8$

Finding the equation using points



A line segment AB is the diameter of a circle, where A and B have coordinates $(5,8)$ and $(-7,4)$ respectively. Determine the equation of the circle.

Hint: What two things do we need to use the circle formula?

?

Test Your Understanding

Edexcel C2 Jan 2005 Q2

The points A and B have coordinates $(5, -1)$ and $(13, 11)$ respectively.

(a) Find the coordinates of the mid-point of AB.

(2)

Given that AB is a diameter of the circle C,

(b) find an equation for C.

(4)

a) ?

b) ?

Completing the square

When the equation of a circle is in the form $(x-a)^2+(y-b)^2=r^2$, we can instantly read off the centre (a,b) and the radius r.
But what if the equation wasn't in this form?

Find the centre and radius of the circle with equation $x^2+y^2-6x+2y-6=0$

Hint: Have we seen a method in a previous chapter that allows us to turn a x^2 term and a x term into a single expression involving x?

?

Textbook Note: There's a truly awful method, initially presented in the textbook, that allows you to find the centre/radius without completing the square. Don't even contemplate using it.

Further Example

Edexcel C2 June 2012 Q3a,b

The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

(a) Find the coordinates of the centre of C.

(3)

(b) Show that $r = 5$

(2)

?

Exercise 6C

Pearson Pure Mathematics Year 1/AS
Page 119-120

Extension:

1 [MAT 2009 1B] The point on the circle $x^2 + y^2 + 6x + 8y = 75$ which is closest to the origin, is at what distance from the origin?

?

2 [MAT 2007 1D] The point on the circle $(x-5)^2 + (y-4)^2 = 4$ which is closest to the circle $(x-1)^2 + (y-1)^2 = 1$ has what coordinates?

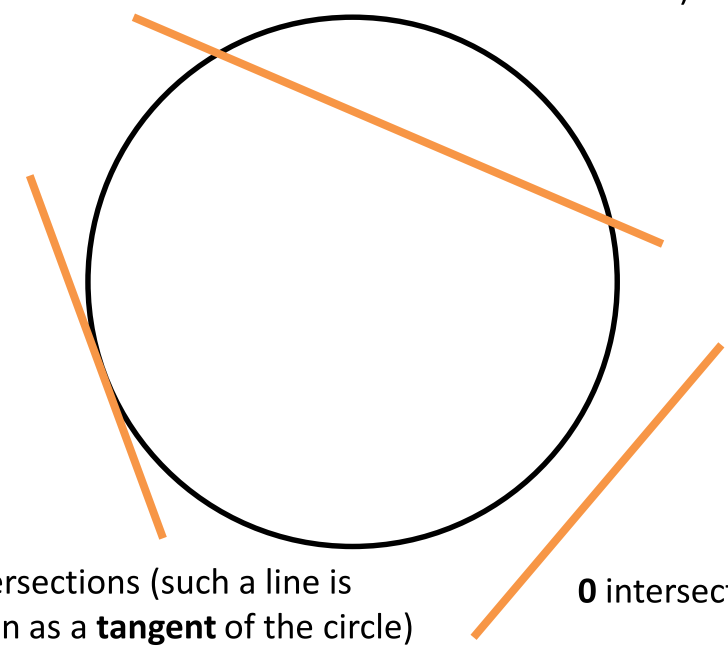
?

3 [MAT 2016 1I] Let a and b be positive real numbers. If $x^2 + y^2 \leq 1$ then the largest that $ax + by$ can equal is what? Give your expression in terms of a and b.

?

Intersections of Lines and Circles

Recall that to consider the **intersection of two lines**, we attempt to solve them **simultaneously** by substitution, potentially using the **discriminant** to show that there are no solutions (and hence no points of intersection).



Show that the line $y = x + 3$ never intersects the circle with equation $x^2 + y^2 = 1$.

?

Test Your Understanding

Find the points of intersection where the line $y = x + 6$ meets $x^2 + (y-3)^2 = 29$.

?

Using an algebraic (and not geometric) method, determine the k such that the line $y = x + k$ **touches** the circle with equation $x^2 + y^2 = 1$.

?

Exercise 6D

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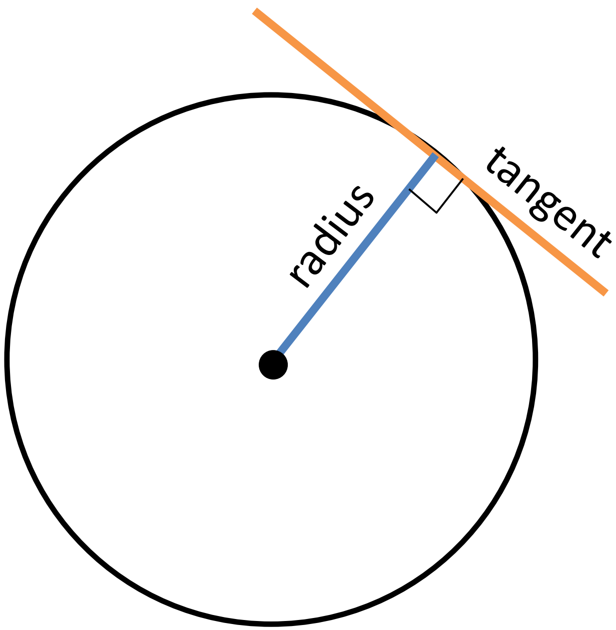
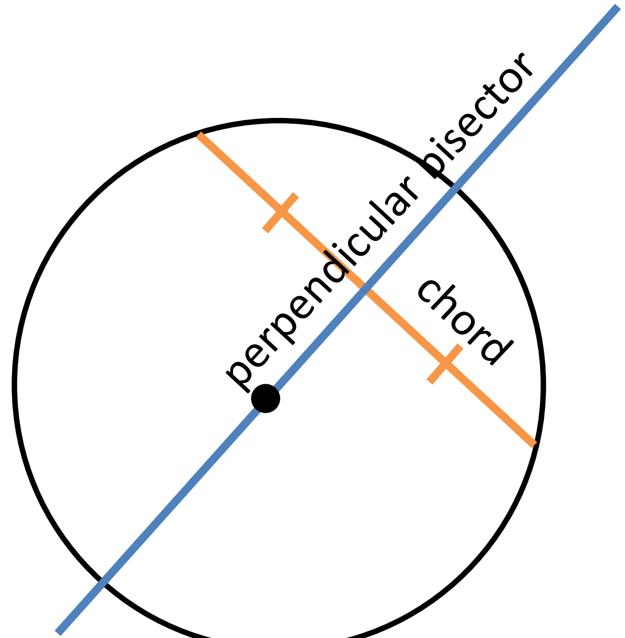
Page 122

Tangents, Chords, Perpendicular Bisectors

There are two circle theorems that are of particular relevance to problems in this chapter, the latter you might be less familiar with:



The tangent is perpendicular to the radius (at the point of intersection).



The perpendicular bisector of any chord passes through the centre of the circle.

Why this will help:
If we knew the centre of the circle and the point of intersection, we can easily find the gradient of the radius, and thus the gradient and hence equation of the tangent.

Why this will help:
The first thing we did in this chapter is find the equation of the perpendicular bisector. If we had two chords, and hence found two bisectors, we could find the point of intersection, which would be the centre of the circle.

Examples

Note that the GCSE 2015+ syllabus had questions like this, except with circles centred at the origin only.

The circle C has equation
$(x-3)^2+(y-7)^2=100$.

a) Verify the point $P(11,1)$ lies on C.
b) Find an equation of the tangent to C at the point P, giving your answer in the form $ax+by+c=0$

?

A circle C has equation

$$(x-4)^2+(y+4)^2=10$$

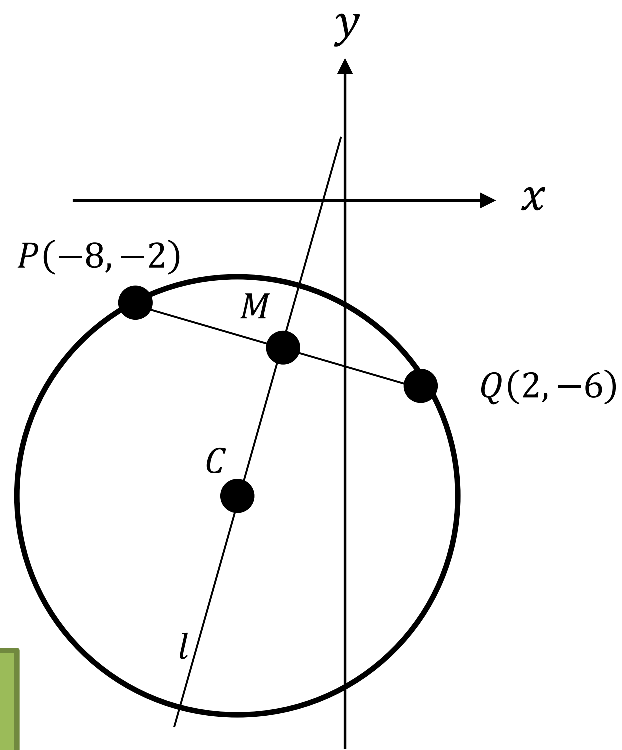
The line l is a tangent to the circle and has gradient -3. Find two possible equations for l, giving your answers in the form $y=mx+c$.

?

Determining the Circle Centre

The points P and Q lie on a circle with centre C, as shown in the diagram. The point P has coordinates $(-8,-2)$ and the point Q has coordinates $(2,-6)$. M is the midpoint of the line segment PQ.
The line l passes through the points M and C.

a) Find an equation for l.
b) Given that the y-coordinate of C is -9:
  i) show that the x-coordinate of C is -5.
  ii) find an equation of the circle.



a

?

ou're not asked
n a particular
as it is.

b

?

c

?

Test Your Understanding

A circle has centre $C(3,5)$, and goes through the point $P(6,9)$. Find the equation of the tangent of the circle at the point P, giving your equation in the form $ax + by + c = 0$ where a, b, c are integers..

?

A circle passes through the points $A(0,0)$ and $B(4,2)$. The centre of the circle has x value -1. Determine the equation of the circle.

?

Exercise 6E

Pearson Pure Mathematics Year 1/AS
Pages 126-128

Extension:

1 *[MAT 2012 1A]* Which of the following lines is a tangent to the circle with equation $x^2 + y^2 = 4$?

A) $x + y = 2$
B) $y = x - 2\sqrt{2}$
C) $x = \sqrt{2}$
D) $y = \sqrt{2} - x$

?

Mark scheme on next slide.

2 *[AEA 2006 Q4]* The line with equation $y = mx$ is a tangent to the circle C_1 with equation $(x+4)^2 + (y-7)^2 = 13$
(a) Show that m satisfies the equation $3m^2 + 56m + 36 = 0$
The tangents from the origin O to C_1 touch C_1 at the points A and B.
(b) Find the coordinates of the points A and B.
Another circle C_2 has equation $x^2 + y^2 = 13$. The tangents from the point $(4, -7)$ to C_2 touch it at the points P and Q.
(c) Find the coordinates of either the point P or the point Q.
Mark scheme on next slide.

(This is not a tangent/chord question but is worthwhile regardless!)

3 *[STEP 2005 Q6]*
(i) The point A has coordinates $(5,16)$ and the point B has coordinates $(4, -4)$. The variable P has coordinates (x, y) and moves on a path such that $AP = 2BP$. Show that the Cartesian equation of the path of P is $(x+7)^2 + y^2 = 100$.
(ii) The point C has coordinates $(a, 0)$ and the point D has coordinates $(b, 0)$. The variable point Q moves on a path such that $QC = k \times QD$, where $k > 1$.
Given that the path of Q is the same as the path of P, show that
$$\frac{a+7}{b+7} = \frac{a^2+51}{b^2+51}$$
Show further that $(a+7)(b+7) = 100$, in the case $a \neq b$.

Mark Scheme for Extension Question 2

(a)	?
(b)	?
(c)	?

Mark Scheme for Extension Question 3

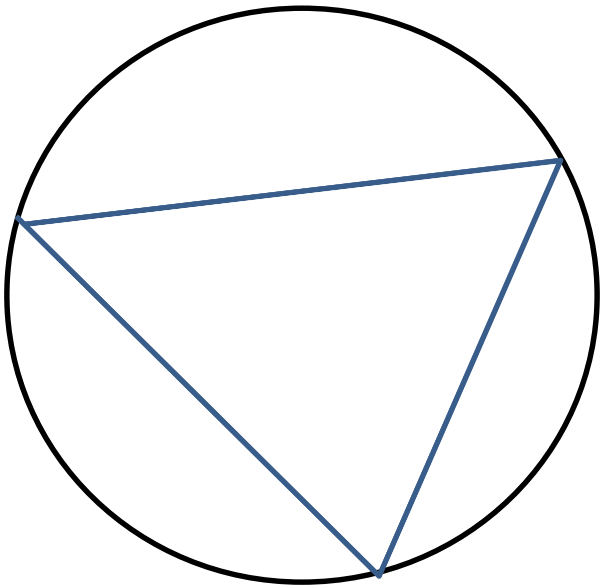
(i)

?

(ii)

?

Triangles in Circles

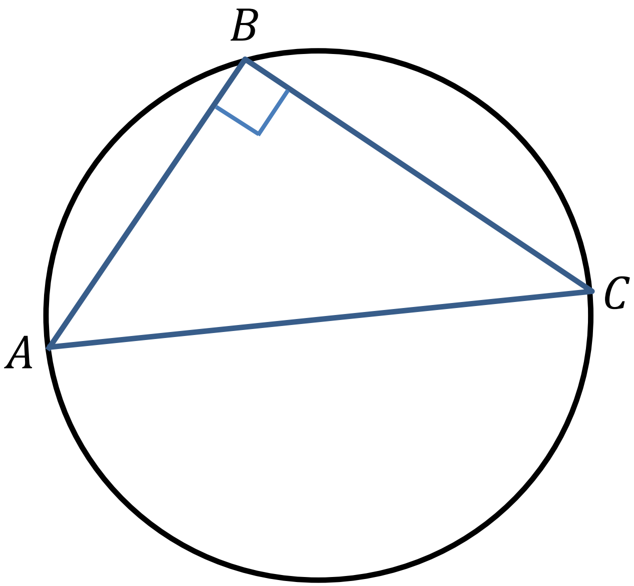


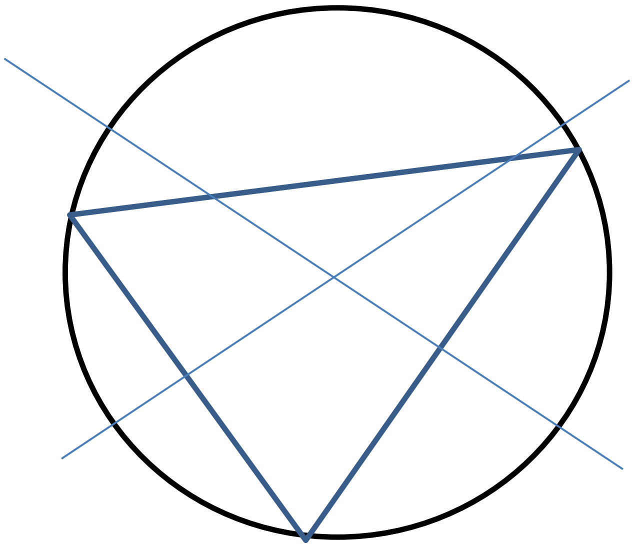
We'd say:

- The triangle **<u>inscribes</u>** the circle.
  (A shape inscribes another if it is inside and its boundaries touch but do not intersect the outer shape)

- The circle **<u>circumscribes</u>** the triangle.
- If the circumscribing shape is a circle, it is known as the **<u>circumcircle</u>** of the triangle.
- The centre of a circumcircle is known as the **<u>circumcentre</u>**.

Triangles in Circles





If $\angle ABC = 90°$ then:

?

Similarly if AC is the diameter of a circle:

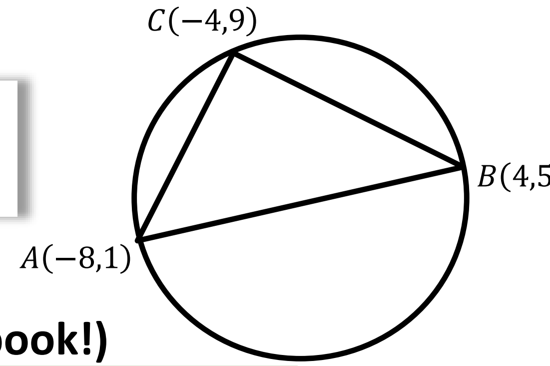
?

Given three points/a triangle we can find the centre of the circumcircle by:

?

Example

[Textbook] The points $A(-8,1), B(4,5), C(-4,9)$ lie on a circle.
a) Show that AB is a diameter of the circle.



Method 1:

?

Method 2: (not in textbook!)

?

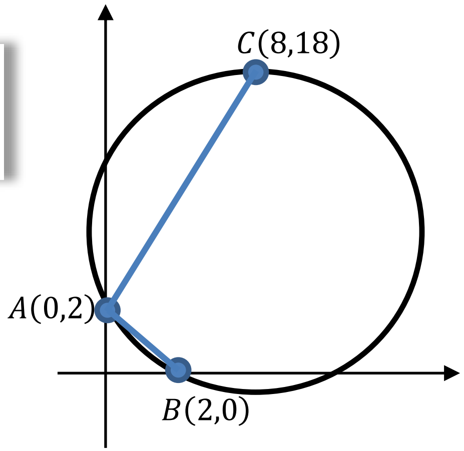
b) Hence find the equation of the circle.

?

Example

The points $A(0,2)$, $B(2,0)$, $C(8,18)$ lie on the circumference of a circle. Determine the equation of the circle.

?



Exercise 6F

Pearson Pure Mathematics Year 1/AS
Pages 131-132

Extension:

1 *[STEP 2009 Q8 Edited]* If equation of the circle C is $(x-2t)^2+(y-t)^2=t^2$, where t is a positive number, it can be shown that C touches the line $y=0$ as well as the line $3y=4x$.

Find the equation of the incircle of the triangle formed by the lines $y=0$, $3y=4x$ and $4y+3x=15$.
Note: The incircle of a triangle is the circle, lying totally inside the triangle, that touches all three sides.

?